(Student Paper) Attitude Dynamics and Stability of Solar Sails During Deployment

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The attitude dynamics of a solar sail spacecraft during deployment are critically important for overall mission success. To model these dynamics, equations of motion governing the deployment of a solar sail are developed and the stability of the predicted motion is evaluated. For this purpose, an expression is derived to calculate the principal moments of inertia as the solar sail is deployed. A typical 40 m x 40 m square solar sailcraft is considered with a center of mass vs. center of pressure (cm/cp) offset of 0.25% of the sail edge length, boom deployment rate of 2.5 cm/s, and maximum incident solar radiation pressure force density of $8.33 \times 10^{-6}$ N/m$^2$. The dynamic simulations show that the sailcraft exhibits an exponential decay in the spin rate about its roll axis (perpendicular to the sail membrane). This behavior is accompanied by an increase in the pointing error of the sail membrane normal vector (i.e., thrust vector). Variations in the baseline parameters given above will alter the final pointing error. It is shown that uncertainty in the cm/cp offset produces the most severe increase in the pointing error. Adjustment of the boom deployment rate (and thus, the total time required for deployment) also significantly affects the pointing error induced by the deployment process. It is observed that the final pointing attitude error in most cases is significantly larger than the typical sailcraft specification of ±1 deg misalignment of the thrust vector. The capabilities of the attitude control system and the deployment parameters (e.g., initial spin rate, boom/sail deployment rate, and initial pointing attitude) must then be optimized to assure satisfactory deployment.

Nomenclature

$(b_i, b_j, b_k)$ = arbitrary vectors
$D_{boom}$ = solar sail boom diameter
$\{e_1, e_2, e_3\}$ = unit basis vectors, global body-fixed coordinates
$\{e'_1, e'_2, e'_3\}$ = unit basis vectors, local body-fixed coordinates
$F$ = magnitude of resultant solar radiation pressure force
$H$ = angular momentum vector about sailcraft center of mass
$I$ = inertia dyadic about sailcraft center of mass
$I_{ij}$ = scalar component of inertia dyadic
$(I'_{11B}, I'_{22B}, I'_{33B})$ = moments of inertia of solar sail boom, local body-fixed coordinates
$(I'_{11S}, I'_{22S}, I'_{33S})$ = moments of inertia of solar sail membrane, local body-fixed coordinates
$(I_{11}, I_{22}, I_{33})$ = principal moments of inertia, global body-fixed coordinates
$L_{boom}$ = length of deploying solar sail boom
$L_{corr}$ = corrected inboard edge length of deploying solar sail quadrant to reflect actual sail area
$m_{bus}$ = mass of spacecraft payload
$m_{craft}$ = mass of entire solar sail spacecraft
$m_{B}$ = mass of stowed solar sail boom
$m_{S}$ = mass of stowed solar sail membrane
$m_{tip}$ = solar sail boom tip mass representing attitude control system hardware

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The attitude dynamics and stability of a solar sail spacecraft during deployment are of great interest since the deployment process itself has a large effect on the overall stability and control of the vehicle. Solar sailcraft dynamics are very complex and nonlinear due to the highly flexible sail membrane, modal interactions among sail components, and contributions from multibody dynamics. In order to advance the Technology Readiness Level (TRL) it is critical that static and dynamic stability of the sailcraft be ensured, with or without active control, during all phases of its operation including the deployment process.

Solar sails, more specifically square solar sailcraft, have been selected by the NASA In-Space Propulsion (ISP) program to undergo system development in order to fulfill near-term flight validation missions. This advance in solar sail technology has produced the need for both revolutionary software and hardware to analyze and construct feasible solar sail prototypes. New tools and methods have been produced to aid in the complex numerical modeling of such gossamer spacecraft. In conjunction with these advancements, there are also prototype solar sailcraft being fabricated and tested under the sponsorship of the ISP program. Successful deployment of a non-spinning solar sail in a thermal vacuum has also been achieved with these designs. Through these projects, much research has been done on the behavior of the sail membrane (e.g., wrinkling) during deployment and on the attitude dynamics and control of a fully deployed solar sail. However, the attitude dynamics of a spin stabilized solar sail during deployment have not yet been investigated.

The solar sailcraft model developed in this paper addresses the attitude dynamics that are induced when the sail booms and membrane are deployed. The first step in this process is to develop equations of motion which describe the dynamics of the spacecraft deployment process. These equations of motion must account for the time-dependency of the area of the deployed sail membrane, spacecraft moments of inertia (which must also be approximated as no previously published data exists), forces generated by solar radiation pressure, and any other significant external disturbances. The stability of the resulting nonlinear attitude dynamics must then be explored by monitoring the temporal roll, pitch, and yaw behavior of the vehicle in both the body-fixed and inertial reference frames.

The following section briefly discusses various aspects of a typical solar sail deployment process, including the characteristics of a few proposed near-term flight validation missions. Equations of motion governing the deployment process are then derived by considering the conservation of angular momentum. Subsequently, the expressions which estimate the principal moments of inertia of the entire sailcraft are given. Lastly, results of the solar sailcraft deployment simulations are presented and characteristics of the predicted dynamics are discussed with emphasis placed on the attitude stability (i.e., the extent to which the sail membrane normal vector deviates from the initial reference direction).

II. Solar Sail Deployment Characteristics

As mentioned previously, much research has been done to characterize the rigid body attitude dynamics of a fully deployed solar sail. Attitude stability of square solar sails (Fig. 1), which are most likely to be utilized in the near future, has also been studied extensively in Refs. 11-16. Several effective attitude control methods have been developed for these types of sails including spin stabilization, center of mass (cm) versus center of pressure (cp)
location manipulation, control vanes at the boom tips, thrusters, etc. However, the attitude dynamics of a solar sailcraft during the deployment process have not been reported previously in the literature.

![Figure 1. Typical square solar sail.](image1)

For any given space orbit, a number of external forces and torques (such as solar radiation pressure, aerodynamic drag, and gravity gradient torques) must be considered during deployment. However, several proposed near-term flight validation mission orbits are sufficiently far from the Earth so that the disturbance torques from solar radiation pressure (SRP) are dominant.\(^\text{17}\) Orbits such as these would allow for tailoring of the initial sailcraft pointing attitude (orientation of the sail membrane normal vector) to exclusively accommodate a desired amount of resultant force generated by SRP. One method to minimize the influence of SRP is to require that sail deployment occur edge-on to the Sun to reduce the amount of photons incident to the sail membrane.\(^\text{18}\) Thus the magnitude of the external disturbance torque from SRP would be minimized, as would the possibility for instabilities induced by sail membrane deployment asymmetries (e.g., uncertain cm/cp offset). It is, however, not clear if this sequence is optimum since the solar sail would be statically stabilized by the incident SRP in a sun-pointing deployment. This choice of initial pointing attitude must be made by comparing the dynamic response in both cases.

Even if the deployment process (whose typical steps are illustrated in Fig. 2) were to take place edge-on to the Sun, this would only minimize the influence of external disturbances. The dynamics of the deploying vehicle itself may still be unstable even with minimal external influence. Proposed methods of addressing total vehicle stability during deployment include spin stabilization,\(^\text{11}\) and the dynamic characteristics of such a spin stabilized deployment are investigated in this paper.

![Figure 2. Typical square sailcraft deployment process.](image2)

### III. Equations of Motion Governing Deployment

To accurately assess the stability of a deploying sailcraft, equations of motion must be developed that describe the complex behavior of the vehicle during that process. These equations must include the time-dependency of a number of quantities (such as sail membrane area, resultant solar radiation pressure, and moments of inertia) in the derivation. For this purpose, Euler’s equations of rotational motion are modified to describe the deployment dynamics of a spinning solar sailcraft. It is assumed that the inertia of the sailcraft changes slowly with respect to time so that no vibrations are induced. That is, any structural effects due to the flexibility of the vehicle components are ignored. The change in geometry of the solar sail during deployment dictates the principal moments of inertia (along with their corresponding time-derivatives) and the total amount of deployed sail membrane which can reflect
SRP. Given these assumptions, and considering a body-fixed coordinate system described by the basis vectors \( \{ \mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3 \} \) whose origin lies at the center of mass of the sailcraft, the following equation can be written which expresses the conservation of angular momentum:

\[
M = \dot{H} = \frac{d}{dt}(I \cdot \omega)
\]  

(1)

Evaluating the time derivative on the right hand side of the equation produces the following:

\[
M = I \cdot \dot{\omega} + \dot{I} \cdot \omega
\]

(2)

The time derivative of the inertia dyadic can be expanded and written as a combination of three components:

\[
M = I \cdot \dot{\omega} + \left[ \omega \times I - I \times \omega + \sum_{i=1}^{3} \sum_{j=1}^{3} \dot{I}_{ij} \mathbf{e}_i \mathbf{e}_j \right] \cdot \omega
\]

(3)

Observe that a dyadic \( \{ \mathbf{b}_i \mathbf{b}_j \} \) has the following associative property:

\[
\{ \mathbf{b}_i \mathbf{b}_j \} \cdot \mathbf{b}_k = \mathbf{b}_i \{ \mathbf{b}_j \cdot \mathbf{b}_k \}
\]

(4)

For purposes of evaluation, the components of the angular velocity vector in the body-fixed reference frame are:

\[
\omega = \omega_1 \mathbf{e}_1 + \omega_2 \mathbf{e}_2 + \omega_3 \mathbf{e}_3
\]

(5)

Examining terms on the right hand side of Eq. (3), the following result is noted:

\[
(I \times \omega) \cdot \omega = 0
\]

(6)

Now, utilizing Eqs. (4), (5), and (6) produces the following simplified and condensed form of Eq. (3):

\[
M = I \cdot \dot{\omega} + (\omega \times I) \cdot \omega + \dot{I}_{11} \omega_1 \mathbf{e}_1 + \dot{I}_{22} \omega_2 \mathbf{e}_2 + \dot{I}_{33} \omega_3 \mathbf{e}_3
\]

(7)

where the body axes defined by the basis vectors \( \{ \mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3 \} \) must be aligned with the principal axes of the sailcraft. This allows the products of inertia to vanish, that is \( I_{ij} = 0 \), for \( i \neq j \). Written in scalar component form, the following equations describe the sailcraft deployment dynamics:

\[
M_1 = \dot{I}_{11} \omega_1 + (I_{33} - I_{22}) \omega_2 \omega_3 + \dot{I}_{11} \omega_1
\]

(8a)

\[
M_2 = \dot{I}_{22} \omega_2 + (I_{11} - I_{33}) \omega_1 \omega_3 + \dot{I}_{22} \omega_2
\]

(8b)

\[
M_3 = \dot{I}_{33} \omega_3 + (I_{22} - I_{11}) \omega_1 \omega_2 + \dot{I}_{33} \omega_3
\]

(8c)

It is important to note that Eqs. (8a,b,c) are similar to the familiar Euler equations for a rotating rigid body, with the exception of the addition of the last term on the right hand side of the equation. This supplemental term accounts for the variation in the inertia of a body over time. This term is omitted in rigid body analyses since the spatial distribution of mass (and thus the moment of inertia) is most often constant with respect to time. However, as a solar sail deploys, its moments of inertia are altered and this variation in inertia is characterized by a certain time rate-of-change whose effects must be included.

In the following analysis, the roll axis (\( \mathbf{e}_1 \)) is perpendicular to the sail membrane and the pitch(\( \mathbf{e}_2 \))/yaw(\( \mathbf{e}_3 \)) axes are the orthogonal transverse axes. See Fig. 1 for an illustration of the body-fixed coordinate axes. Considering the equations of motion for the deployment of a rotating sailcraft (given above), a number of simplifications can be
made to ease the complexity of the dynamic simulations. First, it is assumed that the moments of inertia about the pitch and yaw axes are equal ($I_{22} = I_{33}$ from symmetry). It is also assumed that, during deployment, the sail panels remain flat and the SRP force vector is aligned parallel to the roll axis and through the center of pressure of the sail. This signifies that there is no SRP disturbance torque generated about the roll axis and that the magnitude of the disturbance torque about the pitch and yaw axes is at a maximum. Equations (8a,b,c) can then be rewritten as follows:

$$0 = I_{11} \dot{\omega}_1 + \dot{I}_{11} \omega_1$$

$$M_2 = I_{22} \dot{\omega}_2 + (I_{11} - I_{33}) \omega_1 \omega_3 + \dot{I}_{22} \omega_2$$

$$M_3 = I_{33} \dot{\omega}_3 + (I_{22} - I_{11}) \omega_1 \omega_2 + \dot{I}_{33} \omega_3$$

For ease of simulation, it is then assumed that the orientations of the pitch and yaw body axes are chosen such that $M_2 = 0$ and $M_3 = \varepsilon F$ where $\varepsilon$ is the cm/cp offset and $F$ is the maximum magnitude of the SRP force. The equations of motion then reduce to:

$$0 = I_{11} \dot{\omega}_1 + \dot{I}_{11} \omega_1$$

$$0 = I_{22} \dot{\omega}_2 + (I_{11} - I_{33}) \omega_1 \omega_3 + \dot{I}_{22} \omega_2$$

$$\varepsilon F = I_{33} \dot{\omega}_3 + (I_{22} - I_{11}) \omega_1 \omega_2 + \dot{I}_{33} \omega_3$$

In Eq. (10c), the magnitudes of both $\varepsilon$ and $F$ change during deployment as the geometry of the sail changes. For a baseline 40m square sailcraft, the cm/cp offset is assumed to be 0.25% of the square sail edge length and the magnitude of SRP force is given by $8.33 \times 10^{-6}$ N/m$^2$ multiplied by the area of the deployed sail membrane. The effects of variation in these parameters are presented in Section V. Rearranging Eqs. (10a,b,c) and solving for the angular acceleration leads to the following system of governing ordinary differential equations in the body-fixed reference frame:

$$\dot{\omega}_1 = \left( \frac{I_{11}}{I_{11}} \right) \omega_1$$

$$\dot{\omega}_2 = \left( \frac{I_{33} - I_{11}}{I_{22}} \right) \omega_1 \omega_3 - \left( \frac{I_{22}}{I_{22}} \right) \omega_2$$

$$\dot{\omega}_3 = \left( \frac{\varepsilon F}{I_{33}} \right) - \left( \frac{I_{22} - I_{11}}{I_{33}} \right) \omega_1 \omega_2 - \frac{I_{33}}{I_{33}} \omega_3$$

Again, it is important to note that each of the equations of motion above (for a spinning square or $e_1$-axisymmetric sailcraft under the influence of SRP) contains a newly added term which accounts for the variation in inertia that the vehicle will experience during the deployment process. The angular velocities $(\omega_1, \omega_2, \omega_3)$ in the body-fixed reference frame can then be transformed to their corresponding components $(\dot{\theta}_1, \dot{\theta}_2, \dot{\theta}_3)$ in an inertial reference frame by means of the following $(\theta_3 \to \theta_2 \to \theta_1)$ Euler angle transformation:

$$\dot{\theta}_1 = \omega_1 + (\omega_2 \sin \theta_1 + \omega_3 \cos \theta_1) \tan \theta_2$$

$$\dot{\theta}_2 = \omega_2 \cos \theta_1 - \omega_3 \sin \theta_1$$

$$\dot{\theta}_3 = (\omega_2 \sin \theta_1 + \omega_3 \cos \theta_1) / \cos \theta_2$$

To solve Eqs. (11a,b,c) and (12a,b,c), a Runge-Kutta method of medium (fourth-fifth) order is employed within the MATLAB 7.0 environment (MATLAB ordinary differential equation solver $\to$ ode45).

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IV. Time-Dependent Principal Moments of Inertia

To approximate the principal moments of inertia of the sailcraft, the vehicle is idealized as a system of geometric shapes: four triangular sails, four rectangular booms, a point mass (payload) attached to the sail via an instrument boom, and point masses at the boom tips which represent hardware (spreader bars, control vanes, etc.) used for attitude control of the fully deployed sail. The body-fixed principal axes \{e_1, e_2, e_3\} are defined by the normal to the sail surface which passes through the geometric center of the square sail area, and two orthogonal axes parallel to the boom longitudinal directions and passing through the center of mass of the sailcraft. Inertias for each vehicle component are calculated with the well known formulas:

\[
I_{11} = \int \left( (x'_2)^2 + (x'_3)^2 \right) dm \\
I_{22} = \int \left( (x'_1)^2 + (x'_3)^2 \right) dm \\
I_{33} = \int \left( (x'_1)^2 + (x'_2)^2 \right) dm
\]

with respect to a local coordinate system \{e'_1, e'_2, e'_3\} whose orientation is identical to that of the principal coordinate system with the exception that the origin is shifted to the geometric center of the square planar sail area. The various components are then combined and the parallel axis theorem is applied to form expressions for the overall time-dependent moments of inertia in the global (principal) coordinate system. The moments of inertia of a single boom (whose longitudinal direction is the \(e'_3\) direction) in the local coordinate system are given by:

\[
I'_{11B} = \frac{\rho_{\text{boom}}}{3} \left( (L_{\text{boom}} + R_{\text{hub}})^3 - (R_{\text{hub}})^3 \right) \\
I'_{22B} = I'_{11B} \\
I'_{33B} = \frac{\lambda_{\text{boom}}}{12} \left( D_{\text{boom}}^3 \right)
\]

where \(L_{\text{boom}}\) is the length of a deploying boom which increases with time at a constant rate of 2.5 cm/s\(^{19}\) for baseline 40m sailcraft. A boom deployment rate of 1.0 cm/s is also considered in order to assess the effects of variation in the boom deployment rate. Equations (14a,b,c) indicate that \(I'_{33B} \ll I'_{11B}\) during deployment, since the boom length quickly becomes much greater than the boom diameter, or \((D_{\text{boom}})^3 \ll (L_{\text{boom}})^3\). The moments of inertia of a single sail quadrant in the local coordinate system are given by:

\[
I'_{11S} = \frac{\rho_{\text{sail}}}{6} \left( (L_{\text{corr}} + R_{\text{hub}})^4 - (R_{\text{hub}})^4 \right) \\
I'_{22S} = \frac{\rho_{\text{sail}}}{12} \left( (L_{\text{corr}} + R_{\text{hub}})^4 - (R_{\text{hub}})^4 \right) \\
I'_{33S} = I'_{22S}
\]

where \(L_{\text{corr}}\) is the inboard edge length of a deploying sail quadrant, which is also linearly increasing with time at a constant rate determined by the boom deployment rate. The principal inertias of the entire solar sailcraft are then given by:

\[
I_{11} = 4 \cdot I'_{11S} + 4 \cdot I'_{11B} + 4 \cdot m_{\text{tip}} \left( r_{\text{tip}} \right)^2 + 4 \left( R_{\text{hub}} \right)^2 \cdot (m_{\text{SB}} + m_{\text{SS}}) \\
I_{22} = \left( 4 \cdot I'_{22S} + 2 \cdot I'_{22B} + 2 \cdot I'_{33B} + m_{\text{bus}} \left( r_{\text{bus}} \right)^2 + 2 \cdot m_{\text{tip}} \left( r_{\text{tip}} \right)^2 + 2 \left( R_{\text{hub}} \right)^2 \cdot (m_{\text{SB}} + m_{\text{SS}}) \right) - m_{\text{craft}} \left( r_{\text{cm}} \right)^2 \\
I_{33} = I_{22}
\]

Using published component geometry and mass properties of a typical 40m square solar sail design,\(^{18}\) the equations given above predict the following temporal behavior of the principal moments of inertia:
The length of the time scales in Figs. 3 and 4 correspond to the time required for solar sail deployment to proceed from start to finish for the two specified boom rates. To verify the accuracy of the results given in Figs. 3 and 4, the principal moments of inertia for a 40m solar sail in the fully deployed configuration are compared with published data. The inertias calculated above are 2082 kg-m² about the pitch and yaw (ε₂, ε₃) axes and 3942 kg-m² about the roll (ε₁) axis, whereas the corresponding published values are 2171 kg-m² and 4340 kg-m² (4.1% and 9.2% error, respectively). It should be noted that Eqs. (16a,b,c) are approximations and a more rigorous dynamic model is necessary to predict the complex time dependence of the deploying sailcraft’s inertia with higher accuracy.

V. Simulation Results

The accuracy of the model described in this paper was validated against previously published results for a fully deployed 40m square solar sailcraft. For this purpose, the inertia time-derivatives were set to zero and the following constant values were assigned to moments of inertia, roll rate, and SRP disturbance torque in Eqs. (11a,b,c): (I₁₁, I₁₂, I₁₃) = (6000, 3000, 3000) kg-m², ω₀ = (0.5, 0, 0) deg/s, c = 0.1 m, and F = 0.01 N. Given these parameters, the dynamic model reproduced the previously published results²² shown in Fig. 5. The fully deployed 40m sailcraft exhibits a stable oscillation of the inertial pitch and yaw angles (as shown in Figs. 5b, 5c, and 5d) well within ±1 deg of the reference orientation for a constant roll rate of 0.5 deg/s. This behavior confirms that spin stabilization is a valid method of attitude control for a fully deployed solar sailcraft with similar characteristic parameters.
To simulate the deployment of a 40m x 40m solar sail, the time-dependent principal moments of inertia (and their derivatives as described in Section IV), cm/cp offset, and magnitude of the resultant SRP force are input to the governing system of equations (Eqs. (11a,b,c) and (12a,b,c)) as functions of time. In the baseline 40m sailcraft deployment simulation, the characteristic parameters are as follows: boom deployment rate = 2.5 cm/s, \( \omega_b = (45, 0, 0) \) deg/s, \( \epsilon = 0.25\% \) of sail edge length, and SRP force density = \( 8.33 \times 10^{-6} \) N/m\(^2\). The results of this simulation are shown in Fig. 6, while Figs. 7-10 explore the effects of variation in the initial roll rate, cm/cp offset, SRP force density, and boom deployment rate respectively. The initial condition of \( \Theta_0 = (0, 0, 0) \) deg is applied to the inertial angular displacements in all cases.

The baseline 40m sailcraft behavior shown in Fig. 6a indicates that, after one minute has elapsed, the sailcraft has spun down to slightly less than half of its initial roll rate (\( \omega_1 = 45 \) deg/s). After three minutes, the roll rate drops past 10 percent of its initial value. After slightly more than 5 minutes has elapsed, the roll rate reaches 1 deg/s and proceeds to change very slowly with respect to time (decaying to roughly 0.25 deg/s after 10 minutes has passed, which is halfway through the deployment process). It should be noted that this characteristic exponential decay exhibited by the body-fixed roll rate will occur regardless of the initial value because of the solution to the first order ordinary differential equation given by Eq. (11a). Since there are no previously published figures for a typical deployment process, the initial roll rates utilized in our analyses were selected arbitrarily.
Figure 6. Baseline 40m sailcraft deployment dynamics: boom deployment rate = 2.5 cm/s, $\omega_0 = (45, 0, 0)$ deg/s, $\varepsilon = 0.25\%$ of sail edge length, SRP force density = $8.33 \times 10^{-6}$ N/m$^2$. 
Figure 7. 40m sailcraft deployment dynamics: boom deployment rate = 2.5 cm/s, $\omega_0 = (15, 0, 0)$ deg/s, $\epsilon = 0.25\%$ of sail edge length, SRP force density = $8.33 \times 10^{-6}$ N/m$^2$
Figure 8. 40m sailcraft deployment dynamics: boom deployment rate = 2.5 cm/s, $\omega_0 = (45, 0, 0)$ deg/s, $\epsilon = 2.5\%$ of sail edge length, SRP force density = 8.33x10$^{-6}$ N/m$^2$. 

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Figure 9. 40m sailcraft deployment dynamics: boom deployment rate = 2.5 cm/s, $\omega_0 = (45, 0, 0)$ deg/s, $\varepsilon = 0.25\%$ of sail edge length, SRP force density = $8.33 \times 10^{-7}$ N/m$^2$. 
Figure 10. 40m sailcraft deployment dynamics: boom deployment rate = 1.0 cm/s, $\omega_0 = (45, 0, 0)$ deg/s, $\varepsilon = 0.25\%$ of sail edge length, SRP force density = $8.33 \times 10^{-6}$ N/m$^2$
The baseline 40m sailcraft deployment simulation also shows that the inertial pitch and yaw angular displacements (Figs. 6d and 6e) grow quickly after the 7-9 minute mark. The pitch angle appears to be asymptotically approaching nearly 1.1 deg, but the yaw angle is growing at an increasing rate past 2.5 deg as the deployment process is ending. Generally, the maximum allowable pointing error is ±1 deg. The considerable thrust vector pointing error of nearly 3.0 deg (see also Fig. 6f) at the end of deployment must then be corrected by the on-board attitude control system after it is activated.

Following the baseline simulation, variations of the deployment parameters are studied to assess their effect on pointing error and other results. Decreasing the initial roll rate to 15 deg/s has the logical effect of increasing the pointing error, though only slightly to a final value of roughly -3.5 deg (Figs. 7d, 7e, and 7f). The inertial pitch and yaw angular displacements now increase quickly in the negative direction after 7-9 minutes, though their respective rates of change appear very similar to those exhibited in the baseline case shortly after 10 minutes has elapsed. The pitch angle, however, does not appear to be asymptotically approaching a value in this case. As mentioned previously, the roll rate experiences an exponential decay (Fig. 7a) which is scaled by the initial value in this case of 15 deg/s.

A large cm/cp offset ($e = 2.5\%$ of sail edge length, or fully deployed $e = 1$ m) greatly increases the final pointing error to nearly 27 deg (Figs. 8d, 8e, and 8f). The qualitative behavior of the inertial pitch and yaw angles is similar to that shown in the baseline case, except that the rates of change are elevated so that the pitch angle is now asymptotically approaching 10 deg and the yaw angle is growing past 25 deg. A much higher initial roll rate or some supplemental attitude control methods would have to be employed to mitigate this substantial error.

Significantly decreasing the incident SRP force density to $8.33 \times 10^{-7}$ N/m² (or a fully deployed magnitude of $F = 0.001$ N, as would be possible by dictating an edge-on deployment orientation) has the beneficial effect of lowering the final thrust vector pointing error to approximately 0.25 deg (Figs. 9d, 9e, and 9f). The inertial pitch and yaw angles grow notably after the 7-9 minute mark, but their respective growth rates are significantly lower than those in the baseline case. The final pointing error is also reduced to 10% of the value from the baseline simulation.

Lastly, decreasing the boom deployment rate to 1.0 cm/s (and thus increasing the total time required for full deployment) increases the pointing error to about 11 deg at the end of deployment (Figs. 10d, 10e, and 10f). The pitch and yaw angular displacements now exhibit rapid growth after approximately 24-30 minutes have passed. The pitch angle appears to be approaching 8 deg while the yaw angle is growing past 8 deg. Again, this behavior is qualitatively similar to that displayed in the baseline simulation.

These results illustrate that uncertainty in the cm/cp offset and variation in the boom deployment rate affect the most severe increases in the final pointing attitude error, while the dynamics are less sensitive to changes in the initial roll rate. Also, decreasing the SRP force density has the beneficial effect of significantly decreasing the pointing error. Therefore, in the design of the deployment process, the effects of variation in certain parameters (especially cm/cp offset and boom deployment rate) must be accounted for as they may have a significant effect on the overall stability (i.e., thrust vector pointing error) of the sailcraft.

VI. Concluding Remarks

The attitude stability of a solar sail during deployment has been investigated in order to ensure future mission success. For this purpose, equations of motion which govern the deployment process were derived. Euler’s equations of rotational motion were modified to include a term which accounts for the time-dependence of the moments of inertia of a deploying solar sailcraft. Analytical expressions estimating the principal moments of inertia were also derived, since no previously published data exists for the moments of inertia of a solar sail during deployment. A baseline 40 m x 40 m sailcraft was chosen with the following characteristic parameters: boom deployment rate = 2.5 cm/s, $\omega_0 = (45, 0, 0)$ deg/s, cm/cp offset of 0.25% of the sail edge length, and SRP force density of $8.33 \times 10^{-6}$ N/m². Fully deployed, the last two parameters correspond to $e = 0.1$ m and $F = 0.01$ N. The resulting pointing attitude error at the end of the deployment process is nearly 3.0 deg. A decrease in the initial roll rate to $\omega_0 = (15, 0, 0)$ deg/s only slightly increases the pointing error to nearly 3.5 deg. A large cm/cp offset (of 2.5% of the sail edge length or fully deployed $e = 1$ m) greatly increases the pointing error to almost 27 deg. Reducing the magnitude of the force generated by incident SRP (to fully deployed $F = 0.001$ N) significantly reduces the pointing error to approximately 0.25 deg. Utilizing a slower boom deployment rate of 1.0 cm/s increases the final pointing attitude error to roughly 12 deg. Since the pointing error in most of these cases is larger that the typical sailcraft requirement of ±1 deg maximum misalignment of the thrust vector, the on-board attitude control system and deployment parameters should be optimized to assure satisfactory deployment.
Acknowledgment

This research is supported by the NASA Graduate Student Researchers Program (Grant No. NNM 04-AA03H, Recipient: Brian D. LeFevre) through the Marshall Space Flight Center (MSFC) in Huntsville, Alabama. The authors gratefully acknowledge the guidance provided by Dr. Mark S. Whorton, Branch Chief, Guidance, Navigation, and Mission Analysis, MSFC.

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